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II. Solution by P. S. BERG, B.Sc., Superintendent of Schools, Larimore, N. D.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ELMER SCHUYLER, High Bridge, N. J.; W. H. WILSON, Professor of Mathematics, Geneva College, Beaver Falls, Pa.; and G. B. M. ZERR, A. M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let  $OP=p$ ,  $OC=a$ ,  $OA=b$ ,  $OR=c$ ,  $OI=l$ ,  $IH=m$ ,  $PH=n$ .

The triangles (right angled)  $OPA$  and  $OPI$  are similar, having  $\angle POA$  in common.

$$\therefore p/b=l/p, \text{ or } p^2/b^2=l^2/p^2, \text{ or } p^4/b^2=l^2.$$

$$\text{Similarly, } p^4/a^2=n^2, p^4/c^2=m^2.$$

$$\therefore p^4/a^2 + p^4/b^2 + p^4/c^2 = n^2 + l^2 + m^2 = p^2.$$

$$\therefore 1/a^2 + 1/b^2 + 1/c^2 = 1/p^2.$$

Also solved by J. SCHEFFER, and E. D. SCALES.

111. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Given that the area of a triangle is equal to half the product of two sides and the sine of the included angle, prove that  $\sin(x+y) = \sin x \cos y + \cos x \sin y$ .

I. Solution by W. F. BRADBURY, A. M., Head Master, Cambridge Latin School, Cambridge, Mass., and B. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, O.

Given area  $\triangle = \frac{1}{2}ac \sin B \dots (1)$ .

$$\sin B = \sin[180^\circ - (A+C)] = \sin(A+C).$$

$$\therefore \text{Area } \triangle = \frac{1}{2}ac[\sin(A+C)] \dots (2).$$

Draw  $BD$  perpendicular to  $AC$ .

$$\text{Area } \triangle = \frac{1}{2}BD(AD+DC).$$

But  $BD = c \sin A = a \sin C$ , and  $AD = c \cos A$ , and  $DC = a \cos C$ .

$$\therefore \text{Area } \triangle = \frac{1}{2} \begin{matrix} c \sin A \\ a \sin C \end{matrix} \text{ or } (c \cos A + a \cos C) = \frac{1}{2}(ac \sin C \cos A + ac \sin A \cos C) \dots (3).$$

Putting (2) = (3),  $\frac{1}{2}ac[\sin(A+C)] = \frac{1}{2}ac(\sin A \cos C + \cos A \sin C)$  or  $\sin(A+C) = \sin A \cos C + \cos A \sin C$ .

II. Solution by J. OWEN MAHONEY, B. E., Professor of Mathematics and Science, Carthage High School, Carthage, Tex.; J. W. YOUNG, Columbus, O.; J. SCHEFFER, A. M., Hagerstown, Md.; E. L. SHERWOOD, A. M., Professor of Mathematics, Whitworth College, Miss.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; B. F. SINE, Principal of Normal School, Capon Bridge, W. Va.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; JOHN MACHNIE, A. M., Professor of Latin, University of North Dakota; H. F. STRATTON, Student in Heidelberg University, Tiffin, O.; J. C. NAGLE, C. E., Professor of Civil Engineering and Physics in the Agricultural and Mechanical College of Texas, College Station, Tex.; CHARLES C. CROSS, Libertytown, Md.; and ELMER SCHUYLER, High Bridge, N. J.

PROOF. Consider the triangle  $ACB$ .

The area of  $ACB = \frac{1}{2}ac \sin B = \frac{1}{2}ac \sin(A+C) = \frac{1}{2}hb = \frac{1}{2}h(AD+DC)$ , or

$$\begin{aligned} \sin(A+C) &= \frac{h}{a} \cdot \frac{AD}{c} + \frac{h}{c} \cdot \frac{DC}{a} \\ &= \sin C \cos A + \sin A \cos C. \end{aligned}$$

$$\therefore \sin(x+y) = \sin x \cos y + \cos x \sin y.$$

### III. Solution by the PROPOSER.

Let  $ABC$  and  $CBD$  be the two angles  $x$  and  $y$ . Draw any line perpendicular to  $BC$  meeting  $AB$ ,  $BC$ ,  $BD$  at  $P$ ,  $Q$ ,  $R$ .

Then  $\triangle BPR = \triangle BPQ + \triangle BQR$ .

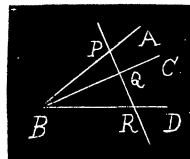
$$2\triangle BPR = BP \times BR \times \sin PBR = BP \cdot BR \cdot \sin(x+y).$$

$$2\triangle BPR = BP \times BQ \times \sin PBQ = BP \cdot BQ \cdot \sin x.$$

$$2\triangle BQR = BQ \cdot BR \cdot \sin y.$$

$$\text{Hence } BP \cdot BR \sin(x+y) = BP \cdot BQ \sin x + BQ \cdot BR \sin y.$$

$$\text{Dividing by } BP \cdot BR, \sin(x+y) = \frac{BQ}{BR} \sin x + \frac{BQ}{BP} \sin y = \cos y \sin x + \cos x \sin y.$$



### CALCULUS.

84. Proposed by C. HORNING, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Find the equation of the curve upon which a given ellipse must roll in order that one of its foci may describe a straight line.

Solution by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Let  $\theta$  be the angle between the major axis of the ellipse and the radius vector from the focus to the point of contact;  $r$  the length of this radius,  $a$ ,  $b$ , semi-axes,  $e$  eccentricity. Then

$$r = \frac{a(1-e^2)}{1-e\cos\theta}.$$

Let the axis of  $x$  be taken parallel to the given line. Then since the point of contact is the instantaneous center, the radius vector will always be perpendicular to the axis of  $x$ , and hence  $r+y=a(1-e)$ ,  $y$  being the ordinate of a point on the required curve. We have also

$$r \frac{d\theta}{dr} = - \frac{dx}{dy}.$$

Eliminating  $r$  and  $\theta$ , we find

$$y - \sqrt{b^2 \left( \frac{dy}{dx} \right)^2 + a^2} = ae.$$

$$\text{Whence } \frac{ae-y}{b} = \sin \left( \frac{x}{b} + \sin^{-1} \frac{ae}{b} \right), \text{ or } y' = b \sin \left( \frac{x'}{b} \right).$$

86. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, 65 Hammond Street, Cambridge, Mass.

Prove that the curve whose normal equals its radius of curvature drawn in an opposite direction, is the catenary  $y = c \cosh(x/c)$ .